JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 48, No. 2, March–April 2011

Using Satellite Constellations for Improved Determination of Earth's Time-Variable Gravity

Brian C. Gunter,* João Encarnação,† Pavel Ditmar,‡ and Roland Klees§
Delft University of Technology, 2629 HS Delft, The Netherlands

DOI: 10.2514/1.50926

The spatiotemporal resolution of the time-variable gravity field models derived from current dedicated gravity field missions is inherently limited by their ground-track coverage. Furthermore, the results are subject to aliasing effects caused by submonthly mass transport signals, such as those caused by atmospheric and ocean processes. To address these issues, this study explores the feasibility of using nondedicated satellite constellations, such as those from commercial communication networks or a low-cost array of custom-built microsatellites, as a complementary data source. The positioning receivers onboard the constellation's satellites would ideally provide a high density of observations in the form of derived accelerations that, while much less accurate than those obtained from dedicated gravity missions, are still sufficient to observe the longest wavelength gravity signals at even subdaily intervals. Using a series of simulated mission scenarios, as well as a limited amount of real-data analysis, it is shown that such constellations, acting either independently or when combined with dedicated gravity field missions, may offer a noticeable improvement in the recovery of the large-scale (greater than 1000 km) high-frequency (less than 1 month) components of the global gravity field.

Nomenclature

$\mathbf{a}$	=	acceleration, m/s^2
$\bar{a}$	=	averaged acceleration, m/s^2
C_{lm}	=	4π-normalized Stokes coefficient for a given degree and order
$\mathbf{e}$	=	vector of constants used to relate averaged and pointwise accelerations
l	=	spherical harmonic degree
m	=	spherical harmonic order
n	=	number of points used in averaging process for accelerations
$\mathbf{V}$	=	gravitational potential, m^2/s^2
x	=	satellite position vector, m
Y_{lm}	=	4π-normalized surface spherical harmonic function for a given degree and order
Δt	=	sampling interval, s
θ	=	angle between two successive line-of-sight range values, rad
μ_E	=	gravitational parameter of Earth, m^3/s^2
ρ	=	intersatellite range, m

I. Introduction

THE gravity field models derived from the Gravity Recovery and Climate Experiment (GRACE) mission data represent the state of the art in observing the time-varying nature of Earth's mass transport processes. The information gathered from these satellites has become invaluable to a number of scientific fields and has helped to quantify processes such as water storage variations at the river basin scale, ice mass loss in the cryosphere, and estimates of sea level rise, to name a few. Many of these results have only been possible as a result of GRACE so, in this respect, the mission has been a tremendous success; however, all missions have their limitations, and improvements to the current GRACE mission concept are already being considered [1,2].

The twin satellites of GRACE fly in a near-polar orbit (altitude ~450 km), in a trailing formation (also called a tandem or leader–follower formation) with a separation of approximately 200 km. Variations in gravity are detected by accurately measuring the intersatellite range of the two satellites, which is done at the micrometer level using a K-band ranging system. That the GRACE mission constitutes only a single instrument pair also has implications on the temporal resolution of the resulting gravity field models. As a general rule of thumb, it takes approximately one month of GRACE data to produce a gravity solution of relatively high spatial resolution and quality: i.e., spherical harmonic degree and order 60 (this translates into a half-wavelength distance of roughly 330 km at the equator) or higher with standard deviations less than 2 cm equivalent water height [3,4]. The reason is that the ground-track coverage needs to be dense enough to adequately observe the smaller features. Earth's gravity is itself changing during this time, as there are many mass transport processes (e.g., atmospheres, continental hydrology, tides, etc.) that have cycles much shorter than one month. The high-frequency (i.e., less than one month) signals created from these short-term processes introduce a large amount of aliasing error into the monthly gravity field models. The current approach for dealing with these errors is to use modeled estimates for these short-period signals and adjust the K-band range measurements accordingly. Such an approach is naturally limited by the accuracy of the models used, and it introduces another potentially large error source into the GRACE gravity field models. Independent of the quality of the background models, what this ultimately demonstrates is that, with only one satellite pair, to get higher spatial resolution, you must sacrifice temporal resolution, and vice versa; the only way to improve both is to increase the number of satellites involved.

One option that could be used to improve the temporal resolution of a future GRACE follow-on mission is to fly multiple pairs of ranging satellites in different orbit planes, as proposed by Bender et al. [5] and others [6–9]. The number of GRACE-like pairs that would need to be flown depends on the accuracy desired, but

Presented as Paper 2009-0191 at the AAS/AIAA Space Flight Mechanics Meeting, Savannah, GA, 8–12 February 2009; received 27 May 2010; revision received 2 December 2010; accepted for publication 7 December 2010.

*Assistant Professor, Physical and Space Geodesy, Delft Institute of Earth Observation and Space Systems, Faculty of Aerospace Engineering, Kluyverweg 1. Senior Member AIAA.

†Ph.D. Candidate, Physical and Space Geodesy, Delft Institute of Earth Observation and Space Systems, Faculty of Aerospace Engineering, Kluyverweg 1.

‡Assistant Professor, Physical and Space Geodesy, Delft Institute of Earth Observation and Space Systems, Faculty of Aerospace Engineering, Kluyverweg 1.

§Professor, Physical and Space Geodesy, Delft Institute of Earth Observation and Space Systems, Faculty of Aerospace Engineering, Kluyverweg 1.

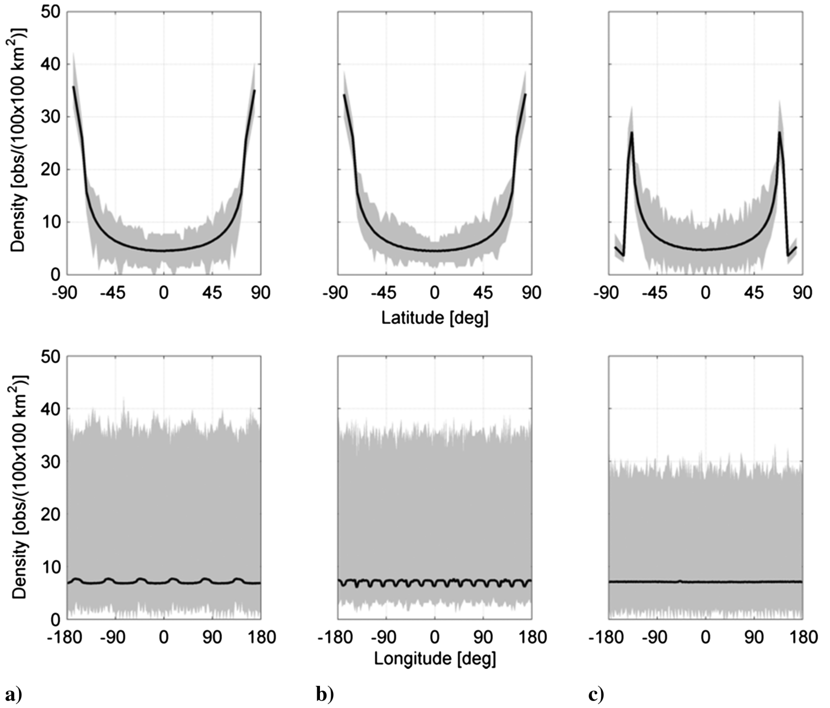

Fig. 1 Mean (solid line) and maximum/minimum (gray) ground-track densities by latitude and longitude for a) the 24-satellite Iridium Next constellation (21 h), b) the GRACE mission (21 days), and c) the F3C/CHAMP constellation (three days).

assuming many pairs are flown, then it would begin to resemble a constellation. While the concept is intriguing, the resources required to build, launch, and support more than one or two GRACE-like satellite pairs are significant, making such proposals academically interesting but not very realistic in the near future. Recognizing this, the focus of this study is to explore alternative ideas that still involve constellations but use satellites from other missions that are not dedicated to gravity field monitoring (i.e., nondedicated satellites). The general concept is to use any available satellites that are, at a minimum, equipped with a high-precision Global Positioning System (GPS) receiver and an attitude determination system. The accelerations acting on the satellite can be derived from the GPS positioning information, in essence, providing low-accuracy gravity measurements. While the measurements would have a lower level of accuracy, the benefit is that there is a constant high-volume stream of globally distributed observations collected, especially in the case where 10s or 100s of satellites are involved. In short, the high density of observations permits the observation of Earth's time-variable gravity field at much shorter time scales than currently possible: i.e., at daily or subweekly intervals. The information from these measurements can then be used as either an independent data set or in combination with a dedicated gravity field mission (e.g., a GRACE follow-on) to help reduce the effects of aliasing and improve the overall accuracy of models generated from such dedicated missions. Such constellations are expected to become a reality in the near future, with plans by the telecommunications industry and radio occultation community to launch the first wave of such GPS-equipped satellites in the next decade: e.g., Iridium NEXT (66 satellites), the Constellation Observing System for Meteorology, Ionosphere and Climate (COSMIC; 6 satellites), COSMIC-II (6 satellites), the Community Initiative for Continuing Earth Radio Occultation project (100 satellites), etc.

This study will focus on the determination of the high-frequency gravity variations: i.e., those which change at timescales less than one month, using the proposed Iridium NEXT constellation as the primary case study. Announced in late 2007,[¶] Iridium NEXT is the next generation of the well-known satellite communication network, with the added benefit that each satellite is expected to carry a secondary scientific payload onboard. While the development of the constellation and associated payloads is still in progress, the current plan is to launch 66 satellites (in six orbit planes, at 780 km altitude, and with an inclination of 86.4°) beginning in 2015, with a scheduled mission lifetime of more than 15 years. Many of these satellites (currently estimated at a minimum of 24) will fly geodetic quality GPS receivers, which could be used for gravity field recovery. As will be seen in the following sections, the case study into the Iridium NEXT constellation nicely demonstrates the feasibility of using nondedicated satellites in the context of time-variable gravity.

In an effort to validate the simulated results of the Iridium NEXT case study, a limited analysis using data collected from the Formosa Satellite Mission No. 3/COSMIC (FORMOSAT-3/COSMIC, or F3C) constellation and GRACE missions will also be presented. The F3C mission is a GPS occultation mission launched in 2006 as part of a joint U.S.–Taiwan collaboration, and it consists of six satellites at an orbit inclination of 72° and an altitude of 800 km. Publicly available orbits for the F3C mission were used to see if the data from these satellites could improve upon current GRACE gravity models. These initial results show that, at current GPS positioning accuracies, the number of satellites in the F3C constellation is insufficient to make such improvements; however, advances expected in the near future in GPS positioning could change this result. In addition, the real-data solutions showed good agreement with results from a corresponding set of simulations, providing further support for the conclusions reached from the Iridium NEXT case study.

II. Methodology

The working principle behind the use of nondedicated satellites for gravity field determination relies on the ability to compute accurate accelerations from precise kinematic orbits provided by GPS tracking data and satellite attitude information. Earlier investigations by Ditmar et al. [10] into the refinement of (static) gravity models from the Challenging Minisatellite Payload (CHAMP) mission discovered that, by properly accounting for the presence of correlated noise in the data (i.e., by using frequency-dependent data weighting), the use of onboard accelerometers becomes essentially unnecessary. In short, the mean gravity fields computed from CHAMP could be derived entirely from the accelerations obtained from the on-board GPS receiver. Further numerical studies [11] supported the

[¶]Data available at http://www.iridium.com/DownloadAttachment.aspx?attachmentID=672 [retrieved 6 Feb. 2011].

conclusion that the effects of nongravitational accelerations acting on the satellite were not the limiting factor in determining gravity models. As a result, the experiments described later do not require that the constellation satellites have an onboard accelerometer; only a GPS receiver and an attitude determination/control system is needed. Note that the satellite attitude is required in order to derive the motion of the spacecraft's center of mass, given the measured position of the GPS antenna.

One of the obvious benefits of using constellations is that the high number of satellites generates a large number of globally distributed measurements. For example, using simulated orbits for the proposed Iridium NEXT (24-satellite case) constellation, it is shown in Fig. 1 that it takes approximately 21 days of GRACE data to generate the same density of observations that the constellation generates in about 21 h. While the observations would be much less accurate than dedicated gravity missions such as GRACE, errors due to temporal aliasing would be significantly reduced. The degree of spatial aliasing would be greatly reduced as well, since this error depends on the number and distribution of the satellites involved, and the constellation would have nearly homogeneous global ground-track coverage.

To examine whether this higher density of observations was sufficient to overcome the lower accuracy of the GPS-derived accelerations, a series of simulations were developed in which the GRACE atmosphere and ocean de-aliasing (AOD) product [12] was used as a realistic time-variable gravity input signal. Derived from global climate data sets, these AOD models are currently used to remove the gravity signals caused by short-term nontidal atmospheric and ocean mass variations from current GRACE level 1B data. It should be noted that the AOD models do not account for all short-term mass transport processes, such as tides and continental hydrology; however, the AOD models do represent the primary signals of interest at the daily to weekly time frames. Neglecting the influence of tides is justified, since these variations are already well understood [13]; therefore, the corresponding tide models are unlikely to be improved by the constellations. The influence of continental hydrology is uncertain, as this occurs across a range of spatial and temporal scales; however, for the purposes of the simulations presented here, precise knowledge of hydrology is not required. This is because the AOD models only need to serve as a realistic (i.e., representative) time-varying gravity field, which they do.

The amplitude of the AOD signal, in terms of the power spectral density (PSD) of geoid height variations, across a range of spatial and temporal scales, is shown in Fig. 2. The figure highlights the fact that the AOD signals have the largest amplitudes at the lower degrees (i.e., less than degree 20, which translates into spatial signals larger than 1000 km), but that these amplitudes can span across all time scales within a month (i.e., from subdaily to 31 days). In short, the figure represents the signal that the current GRACE mission is unable to observe, and which must be removed in the data preprocessing. The AOD models are naturally subject to inaccuracies, as they are

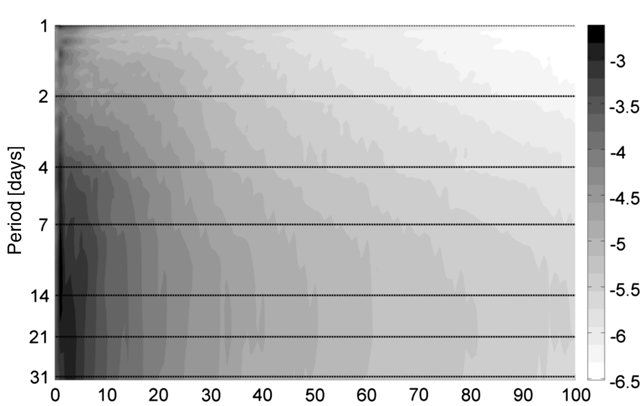

Fig. 2 PSD of the AOD models across varying time frames (1 to 31 days), in decimal orders of magnitude of geoid height [$\log_{10}(\mathrm{m}/\sqrt{\mathrm{Hz}})$].

produced from climate data of varying quality and global distribution. Recent studies [14] have shown that, for a mission such as a GRACE follow-on, or even the current GRACE mission operating at the originally projected baseline performance level, uncertainties in the AOD models could be one of the dominant error sources. If such signals can be observed by the proposed constellations examined in this study, this would translate into improved gravity models, and it is one of the main reasons why the AOD models were chosen as the time-varying signal used in the simulations.

Accelerations were derived for each satellite in the constellation from absolute position data, representative of what might be expected from a GPS receiver, using a realistic colored noise spectrum derived from the earlier studies done on CHAMP mission data [10]. A GRACE-like formation was also simulated, with accuracies similar to those of the actual mission. A series of gravity field models were then computed over time spans ranging from 1 to 31 days for both the individual (i.e., Iridium 24-satellite or GRACE) and combined (i.e., Iridium 24-satellite and GRACE) scenarios. These models were then compared with the actual time-varying AOD signal (i.e., the truth input signal) averaged over the corresponding time interval. By using only the AOD product in the force model, the results highlight what contributions such a constellation might make toward resolving those short-term signals to which GRACE is inherently insensitive.

To evaluate the influence of spatial and temporal aliasing errors for a given constellation scenario required the development of a robust simulation environment. The software developed for this purpose is capable of generating multiple observation types (GRACE-type and single-point accelerations) with realistic noise spectra and in the presence of a constantly varying gravity field.

The errors associated with inaccuracies in the instruments were introduced by corrupting the simulated observations with noise (i.e., measurement noise). Temporal aliasing was evaluated by examining the residual between a time-varying gravity field (the real force model) and a static one (the reference). An inversion of observations created using either measurement noise or temporal aliasing results in a gravity field model that will differ from the time average of the true dynamic model, thereby providing an indication of the magnitude of each error source. The assumption made in this study is that the measurement and temporal aliasing errors are independent, and they can therefore be computed separately and later combined. Although not shown here, differences in the results between cases in which the errors were treated separately or combined showed only small differences and support this claim. Note that since the AOD product is produced in 6 h time intervals, a linear piecewise interpolation scheme was used to compute the time-varying gravity during these intervals. The choice of the static field used is essentially arbitrary, but for this study, the EIGEN-CG03C [15] model was used.

The maximum degree and order of the force model in the simulations was set to 100, which is the maximum resolution of the AOD models. For the Iridium NEXT case studies, spherical harmonic coefficients were estimated only up to degree 45 due to restrictions caused by the polar gap of the Iridium constellation (45 is the maximum degree allowable with a 4° polar gap).

A. Simulated Global Positioning System Accelerations

For the observations related to the constellations, it was assumed that a time series of position estimates would exist, such as a kinematic orbit product derived from an onboard GPS receiver. These positions were simulated for the constellations by integrating the equations of motion of each satellite, using stable Keplerian orbits to keep the constellation geometry unchanged. A pointwise acceleration was then computed at each epoch using a spherical harmonic synthesis of the real gravity field model. Again, the real model in this case is simply a static field (when computing measurement noise error) or a static field that has been altered with the time-varying AOD data (when computing temporal aliasing error). The link between the Stokes coefficients of the gravity model and a pointwise acceleration is straightforward, and it starts with the relationship between the acceleration $\mathbf{a}$ and the gravitational potential $\mathbf{V}$:

$$\mathbf{a} = \nabla \mathbf{V} + \mathbf{a}^{(\mathrm{ng})} \tag{1}$$

$$= \nabla \mathbf{V}_s + \nabla \mathbf{V}_{\mathrm{tv/noise}} + \mathbf{a}^{(\mathrm{ng})} \tag{2}$$

where $a^{(\mathrm{ng})}$ represents the nongravitational accelerations acting on the satellite (these were set to zero for the simulations, under the assumption that the orbit determination process would sufficiently account for them). The separation of the potential gradient into two parts is to highlight the fact that the static component $\nabla \mathbf{V}_s$ is computed separately from the component due to time-variable gravity or measurement noise $\nabla \mathbf{V}_{\mathrm{tv/noise}}$. The relationship between the Stokes coefficients and the gravitational potential at a point defined by the spherical coordinates (r, θ, λ) is described as

$$\mathbf{V}(r, \theta, \lambda) = \frac{\mu_E}{R} \sum_{l,m=0}^{\infty} \bar{C}_{lm} \left(\frac{R}{r}\right)^{l+1} \bar{Y}_{lm}(\theta, \lambda) \tag{3}$$

with $\bar{Y}_{lm}$ representing the 4π-normalized surface spherical harmonic function for a given degree l and order m, $\bar{C}_{lm}$ representing the corresponding Stokes coefficients, and R representing the semimajor axis of a reference ellipsoid.

For the real-data processing scenario, to be described later, a three-point differentiation scheme is used to compute an averaged acceleration from the position estimates. To emulate this for the simulated GPS observations requires that we apply some type of averaging to the series of pointwise accelerations. The averaging is done by applying the filter outlined in Ditmar and van Eck van der Sluijs [16], for which the number of points n involved typically ranges from six to eight, and the degree of the polynomial ranges from 12 to 16 (i.e., $2n$).

An averaged acceleration [i.e., $\bar{a}(t)$] is related to the pointwise accelerations by the expression

$$\bar{a}(t) = \mathbf{e}^T \mathbf{a} \tag{4}$$

where the constant vector $\mathbf{e}$ consists of filter coefficients, which only need to be computed once (again, see Ditmar and van Eck van der Sluijs [16] for further details).

Measurement noise in the GPS observations is simulated by generating colored noise, which realistically reproduces the uncertainty associated with GPS positioning, as determined by experiments with CHAMP mission data [10]. The spectrum of the noise along each component (along track, cross track, and radial) is shown in Fig. 3. A realization of this noise $\sigma(t)_{\mathrm{noise}}$ is added directly to the averaged accelerations to finalize the creation of the simulated GPS observations; that is,

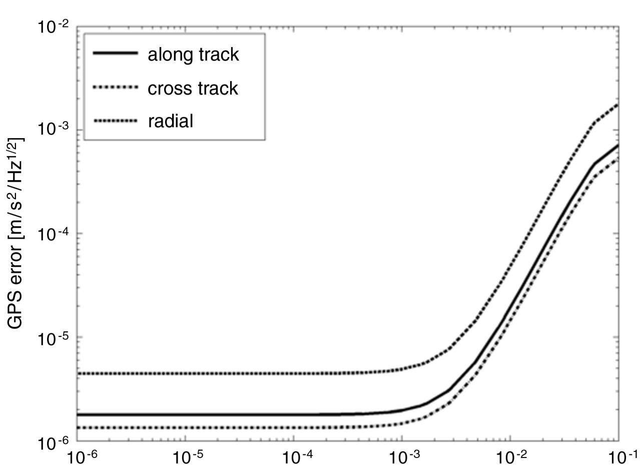

Fig. 3 Square-root PSD of the GPS receiver errors, in terms of absolute accelerations, used in the simulations.

$$\bar{a}(t) = \mathbf{e}^T \nabla \mathbf{V}_s + \sigma(t)_{\mathrm{noise}} \tag{5}$$

In using a measurement noise spectrum derived from CHAMP, we assume that the GPS receiver performance for the constellations will generate orbit position accuracies at essentially the same level as CHAMP, or approximately 2–3 cm. In a study conducted by Iridium to determine the requirements for the altimetry payload expected for the Iridium NEXT constellation [17], the non-time-critical orbit products (i.e., latencies great than one month) are listed as having expected accuracies of less than 2 cm, which fits well with the assumed measurement noise levels used in this study. For the case of temporal aliasing error due to time-variable gravity, the pointwise acceleration of the AOD signal is first added to the static component, and then the result is averaged:

$$\bar{a}(t) = \mathbf{e}^T (\nabla \mathbf{V}_s + \nabla \mathbf{V}_{\mathrm{tv}}) \tag{6}$$

B. Simulated Gravity Recovery and Climate Experiment Observations

To generate simulated GRACE-like observations required different handling, as the observations involve both the absolute and relative positions of the two satellites. For this case, observations were created using the range-combination approach developed by Liu et al. [4,18]. This technique computes an averaged acceleration a_{rc} based on the linear combination of three successive intersatellite line-of-sight (LOS) range measurements collected at time epochs $i-1$, i, and $i+1$, with

$$a_{\mathrm{rc}} = \frac{(\cos\theta_{i-})(\rho_{i-1}) - 2\rho_i + (\cos\theta_{i+})(\rho_{i+1})}{(\Delta t)^2} \tag{7}$$

Here, Δt is the sampling rate (5 s in this study), ρ is the intersatellite range between GRACE satellites A and B at each epoch, and θ_{i+} and θ_{i-} are the LOS angles of the first and third range measurements with respect to the LOS measurement at time i, as shown in Fig. 4.

The values used for the inversion step are the residuals computed by differencing the accelerations derived from the range combinations [Eq. (7)] with the averaged accelerations calculated on the basis of a reference gravity field model [i.e., generating pointwise accelerations and applying Eqs. (5) or (6)] and projected onto the LOS. The inclusion of either temporal aliasing or measurement noise was done by choosing the appropriate force model for the orbit integration. For the measurement noise cases, only a static reference field was used in the force model. Sensor uncertainty in the K-band ranging system was implemented by corrupting the intersatellite ranges with white noise [i.e., altering only ρ of Eq. (7)]. Increasing or decreasing these noise levels has a direct impact on the sensitivity of the GRACE satellites to the gravity variations. Noise was also added to the individual GRACE satellite positions (altering θ but maintaining the same intersatellite range ρ) to emulate realistic orbit errors. For temporal aliasing, perfect ranges and positions were created, but the force model used for the orbit integration was changed from a purely static field to a field that included the time-varying AOD signal.

The stability of the GRACE-like formation was assured by resetting the osculating orbital elements every 6 h to those of a reference (and stable) Keplerian orbit. Any accelerations computed that may have involved the discontinuities present at the extremities of each 6 h arc were discarded.

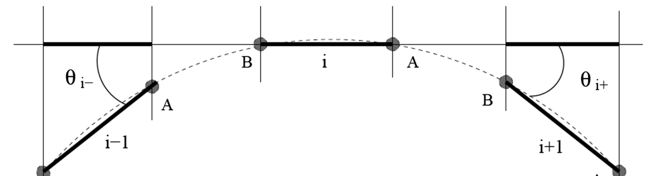

Fig. 4 Illustration of the range-combination approach used to create simulated GRACE measurements.

C. Measurement Noise Scenarios

To better understand the influence that measurement noise in the simulated observations might have on the resulting gravity field models, two different cases were examined. The first, called the high-noise case, represents what is believed to be the accuracy of current spaceborne GPS receivers and the GRACE K-band ranging system. For GRACE, this translates into absolute position errors with standard deviations on the order of 1 cm, relative positioning errors at the 1 mm level, and intersatellite ranging errors of 10 μm. For the constellations, errors in the accelerations were implemented using the PSD distribution of Fig. 3. While this high-noise case represents the current state of the art, there are many advances expected in the near future within the Global Navigation Satellite System and geodesy communities that will likely improve upon these current accuracies. The launch of the next generation of GPS satellites (Block-III) along with the upcoming deployment of the European Galileo system are two such examples. The Galileo system alone promises more accurate broadcast ephemeris, better atmospheric correction capabilities, and improved clocks; plus, the additional number of satellites will help in terms of geometry (reduced dilution of precision) and the number of visible satellites. Similarly, a future GRACE follow-on mission will likely employ a laser-ranging system, which preliminary investigations have already demonstrated can significantly outperform the current K-band system [19]. As such, a second low-noise case was evaluated, for which we have assumed that there will be an order-of-magnitude improvement in the determination of GPS-derived accelerations and intersatellite ranging capabilities in the near future (i.e., the high-noise errors were simply reduced by a factor of 10). While it is recognized that this degree of improvement is likely optimistic, the goal of this low-noise case was to establish a reasonable upper bound of the performance that might be achieved through future technological advancements. As expected, the influence of the measurement noise on the final solution depends on the type and number of observations collected. Figure 5 shows the high- and low-noise measurement levels for the three primary mission scenarios simulated in this study (Iridium, GRACE, and F3C/CHAMP) over two different time spans (1 day and 31 days). The figure also illustrates the size of the measurement noise with respect to other sources of error present in the simulations, namely, spatial and temporal aliasing errors. Note that the measurement noise for the constellations stays approximately the same for both time frames, meaning that the constellations achieve essentially the same level of accuracy at 1 day that they do at 31 days. In contrast, the GRACE-type formation is more sensitive to measurement noise at the 1-day interval, with errors a factor 10 larger than the constellations; but as more observations are collected, this error becomes significantly lower than the constellations at the 31-day mark.

D. Spatial Aliasing

An additional error component is present in the simulations in the form of spatial aliasing. This spatial aliasing is essentially a type of truncation error incurred by the fact that we are only estimating out to degree 20 (F3C/CHAMP) or degree 45 (Iridium), even though there is signal out to degree 100 in the AOD models. To quantify this spatial aliasing error, solutions were estimated out to degree 20 using observations generated from a degree 100 static reference field in the

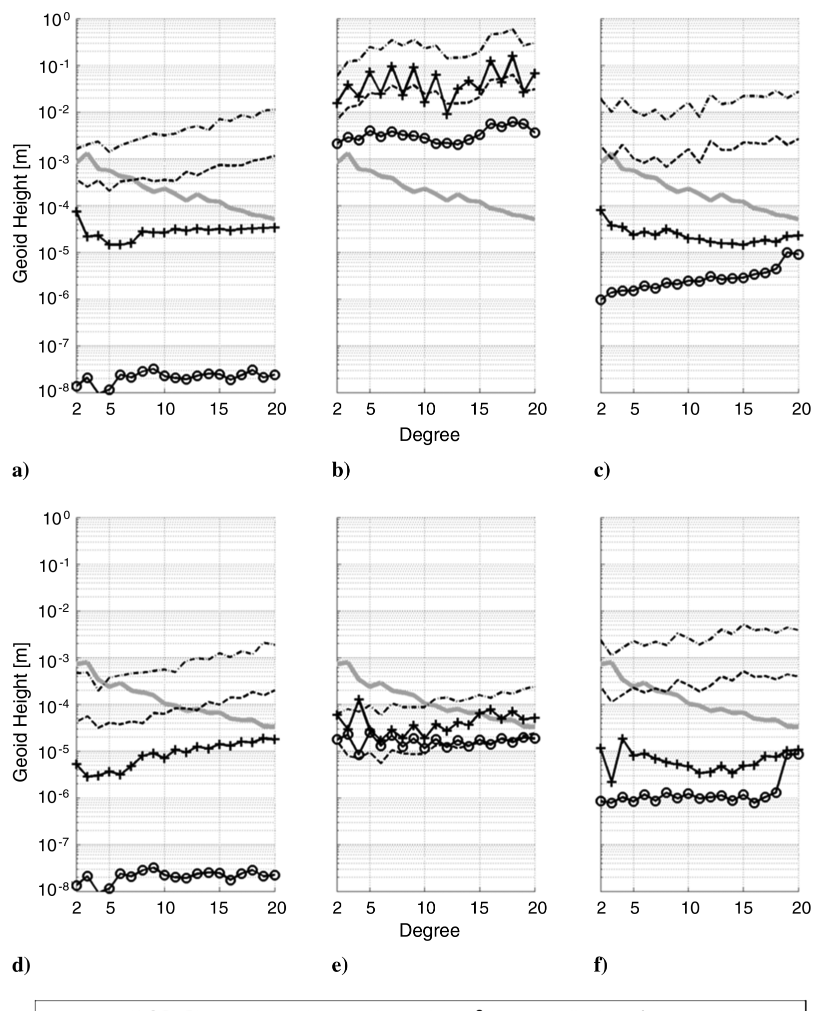

Fig. 5 Degree variance plots of the various noise components present in the following simulations: a and d) Iridium 24-satellite, b and e) GRACE, and c and f) F3C/CHAMP over the 1-day (a–c) and 31-day (d–f) time intervals.

force model. The difference between the estimated and input static reference fields represents the amount of spatial aliasing error present in the solutions, and it is shown in Fig. 5. For the constellations, the error due to spatial aliasing is essentially negligible when compared with the other error sources (temporal aliasing and measurement noise); however, for GRACE, the spatial aliasing is much more significant and can exceed the measurement noise in some instances. Note that, for the experiments involving temporal aliasing (to be described shortly), the solutions are also estimated out to degree 20 or 45, but the observations are generated using a time-varying (non-static) gravity field. For these cases, the calculation of the temporal aliasing error, by nature, includes the component due to spatial aliasing error. Future references to temporal aliasing errors imply that both spatial and temporal aliasing errors are considered.

III. Simulation Results

Focusing first on the performance of the constellations alone, Fig. 6 shows the degree variances of the estimated gravity field model for the high- and low-noise cases for a hypothetical Iridium 24- and 66-satellite constellation over a 31-day time span. As already illustrated in Fig. 5, the error from temporal and spatial aliasing is quite small in both cases, so the error levels shown are almost entirely influenced by the performance of the GPS receivers. For comparison, the 31-day average of the AOD product is also shown. What the figure demonstrates is that, at least for the 31-day case, the 24- and 66-satellite constellations are able to observe some of the AOD signal spectrum (up to degree 15 for the low-noise 66-satellite case). Naturally, more signal is observed as more satellites are added and as the accuracy of the GPS measurements improve. Since many other time frames were evaluated in addition to the 31-day case, the left panel of Fig. 7 shows a more complete picture of the performance of the constellations: in particular, the Iridium 24-satellite low-noise case. The plot shows the solutions expressed in terms of signal-to-noise ratios, created by comparing the computed solution for each time frame with the time-averaged AOD product (i.e., the truth). The plots represent the spatiotemporal accuracy of each scenario, with the vertical (log scale) axis representing the duration of the solution (i.e., one day, two day, etc.), and the horizontal axis represents the spatial resolution achieved in units of spherical harmonic degree. A ratio of 1.0 (delineated by a white line in the plot) or higher implies that the gravity signal can be observed at the given time and spatial resolution, and it is analogous to the point at which the AOD and Iridium curves cross in Fig. 6 (for the 24-satellite low-noise 31-day case).

The next step was to compare the performance of the combined mission scenarios: i.e., using both GRACE and the constellations. The results are shown in Fig. 7c, and they represent the combination of the Iridium 24-satellite low-noise case with the GRACE high-noise case (with both spatial and temporal aliasing error also included). Two important conclusions can be made from this figure. First, the Iridium constellation observes the AOD time-variable signal up through degree 7 for all time scales, where a GRACE-like mission is limited to signals with time scales of approximately four days and higher. More important than the individual performances is the fact the combined solution can observe a larger part of the AOD spectrum. This indicates that the addition of the constellation data was able to reduce the amount of aliasing present in the combined solution, allowing the harmonics at the higher degrees to be estimated more accurately.

Furthermore, the results of Fig. 7 demonstrate a fairly realistic scenario. The GRACE measurements represent those of the current GRACE mission. The low-noise Iridium 24-constellation is also an intermediate option on the side of the constellations, because there are options that are better, such as the low-noise Iridium 66 solution. In fact, the low-noise Iridium 24-constellation, when considering Fig. 6 again (i.e., the error curve falls in the middle of the Iridium 66 band), could also represent a 66-satellite constellation with, say, only a factor of six to seven in improvement in errors, or it could represent any other permutation of satellites from 24–66 with varying levels of improvements: e.g., 45 satellites with a factor 5 improvement in GPS error. The results show that, even in this combination, which could conceivably be realized in a short time frame (less than five years), the combined solution is a noticeable improvement over the GRACE-only solution.

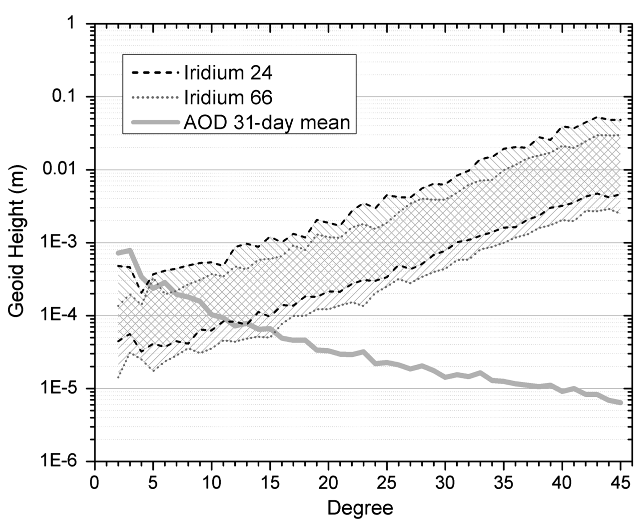

Fig. 6 Degree variances of the total error bands for the 31-day Iridium 24- and 66-satellite constellation simulations using assumptions for high and low measurement noise.

IV. Comparisons with Real Data

The results to this point have primarily relied on simulations, so this section will attempt to validate the predications of these earlier simulations through the processing of genuine mission data. Using an approach similar to that used for the simulations, accelerations were derived from existing F3C orbit data and combined with similar data available for CHAMP, which is also equipped with a GPS receiver. While only a single satellite, the primary benefit of including CHAMP data is that CHAMP's high inclination (87°) helps to fill the (sizeable) gap at the poles left by the F3C satellites.

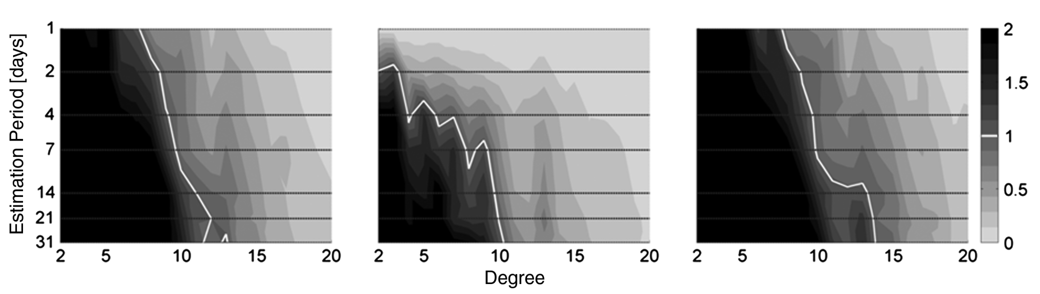

Fig. 7 Signal-to-noise ratios for a) the Iridium 24-satellite low-noise cases, b) the GRACE high-noise cases, and c) the resulting combined solutions for a range of spatial (spherical harmonic degree) and temporal values.

In addition to the real-data analysis, a series of corresponding simulations were also developed to evaluate the reliability of the results from the previous section. The idea being that if the simulated results match closely with those of the real-data processing, then the projections of the Iridium NEXT constellations should be realistic. It should be noted that, while the simulations tried to match the actual F3C scenario, some simplifications were made. For example, the orbit altitude for all F3C satellites were fixed at 800 km and were circular, with the ascending nodes of the individual satellites' orbit planes evenly distributed (i.e., 60° increments). In reality, the altitude and node separation of the various F3C satellites are not constant, but the simplifications made were assumed to be close enough to the actual mission design to permit a reasonable assessment of the constellation. For comparison with the earlier simulations shown for the Iridium constellations, the ground-track density and noise budget plots for the F3C/CHAMP constellation can be found in Figs. 1c and 5c.

A. Real-Data Processing

The processing of the F3C/CHAMP mission data attempted to keep as consistent as possible with that of the simulations so that comparisons between the two could be made. The orbits for the F3C and CHAMP satellites were obtained from the University Corporation for Atmospheric Research COSMIC Data Analysis and Archive Center (CDAAC), which were created using the kinematic approach developed by Švehla and Rothacher [20]. The kinematic approach makes almost exclusive use of the GPS data to determine the satellite orbit, as opposed to the more traditional (reduced) dynamic approach that attempts to model a range of forces (i.e., static gravity, tides, third body, etc.). The stated accuracies of the F3C satellite orbits, based on external validation [21], were found to be at the level of 10 cm in absolute position (equivalent to velocity errors of $0.1\ \mathrm{mm/s}$ three-dimensional rms). This level is somewhat less accurate than the 2–3 cm level accuracy that can be achieved from dedicated gravity field mission satellites, such as GRACE; however, this is primarily due to the fact that the kinematic approach is sensitive to GPS receiver errors (data gaps, spikes, etc.), and the multipath error created by the F3C solar array panels is much larger than GRACE, which does not have such panels [21]. The importance of this difference in orbit accuracy will be shown later. To maintain consistency of the orbit products used for the constellation, it should be noted that the CHAMP orbits used in the analysis, to be presented shortly, were also obtained from CDAAC, with the same level of estimated accuracy as the F3C satellites.

For each satellite in the constellation, the point positions obtained from CDAAC orbits were used to compute a time series of accelerations at a satellite altitude using the three-point differentiation scheme [16]:

$$\bar{a}(t) = \frac{x(t-\Delta t) - 2x(t) + x(t+\Delta t)}{(\Delta t)^2} \tag{8}$$

where $x(t)$ is a component of the satellite position vector at time t and Δt is the data sampling interval (5 s for this study). The acceleration computed in Eq. (8) is an averaged acceleration and not a pointwise acceleration, but it can be related to the Stokes coefficients through the relationships outlined in Eqs. (1–6).

To permit comparisons to the simulations, residual accelerations were computed by removing a static reference field. Additional effects due to processes such as diurnal/semidiurnal tides and solid-Earth tides were also removed. Data weighting was achieved through the application of an optimally determined autoregressive moving average (ARMA) filter [22,23], based on the PSD of the computed residual accelerations. Normally, the ARMA filter is based on the noise PSD; however, for the F3C constellation, the signal-to-noise ratio was not expected to exceed unity so, in this case, the residual accelerations served as an approximation of the noise. The maximum degree and order of the real-data solutions (and corresponding simulation) was also limited to 20.

Most of the analysis for the real-data processing was restricted to the month of August 2006. This month was chosen because it represented a month in which the corresponding GRACE solution was of good quality. That said, it should be emphasized that there is variability in both the GRACE solution quality and in the variation of the AOD signals from month to month. The GRACE accelerations were derived from RL04 L1b data using a range-combination approach similar to that discussed earlier, following the processing standards used to develop the Delft Institute of Earth Observation and Space Systems (DEOS) Mass Transport (DMT-1) series of publicly available GRACE gravity models [4].

B. Results

1. *Simulated Results*

The simulated results of the F3C/CHAMP constellation are shown in Fig. 8, where once again a series of high-noise (Figs. 8a–8c) and low-noise (Figs. 8d–8f) solutions were created over time spans that varied from one day to one month. For each noise case, a corresponding set of GRACE-like measurements were created, and a combined F3C/CHAMP/GRACE solution was generated. It is important to note again that the only time-variable gravity signal used in the simulations was that introduced by the AOD product for August 2006.

The first observation that can be made from this figure is that the high-noise case for F3C/CHAMP is not able to clearly observe any of the AOD signal, as the signal-to-noise ratio is much less than one at all degrees and time frames. The high-noise GRACE-only case is sensitive to some signals up to approximately degrees 7–8, but only those signals with a frequency greater than or equal to four days. The combined F3C/CHAMP/GRACE case for the high-noise case shows little change from the GRACE-only case, again highlighting that at current positioning accuracies, the F3C/CHAMP constellation is not able to sense time-variable gravity signals that are at the size of the AOD signal.

The low-noise case is much different, however, and shows that the F3C/CHAMP constellation is able to observe some of the AOD signal. The lower noise also benefits the GRACE-only case, as seen in the lower middle plot. Most important is the fact that the F3C/CHAMP/GRACE combination shows a noticeable improvement over the GRACE-only case, indicating that the F3C/CHAMP constellation data has a stabilizing effect on the solution and allows the higher degrees to be estimated with greater certainty. The results indicate that a combined low-noise solution should be able to resolve time-variable features up through degree 15 at temporal scales of two days and higher and daily/subdaily variations at spatial scales up to degrees 7–8.

2. *Real-Data Results*

Given the expected insensitivity of the F3C/CHAMP solutions to the AOD signal at current GPS positioning accuracies, the goal of the real-data processing was more to see if the assumptions made in the simulations were at least realistic. A degree and order 20 solution was created from the CDAAC precise orbits for August 2006, and the (square root) degree variance of this solution was compared with the corresponding solutions created from simulation. The results can be seen in Fig. 9, along with the corresponding real-data GRACE solution for the same month. The error bars in the figure for the simulated results indicate the standard deviation from a series of six different noise realizations. For the high-noise case, the comparison with GRACE echoes that of Fig. 8, in that the accelerations derived from F3C/CHAMP constellation are currently not accurate enough to exceed the contribution from GRACE at the 31-day time frame. The combined F3C/CHAMP/GRACE shows very little difference from the GRACE-only solution.

In terms of whether the (high noise) simulated results and the real-data processing are close to each other, we see that at the lower degrees (less than 10) of the two solutions compare rather favorably. Toward the higher degrees, the accuracy of the real-data solution becomes worse than that predicted from the simulation. This can mainly be attributed to the accuracy of the kinematic orbits used in the real-data processing. The stated accuracy of the orbits was at the 10 cm level, whereas the simulations assumed a higher level of

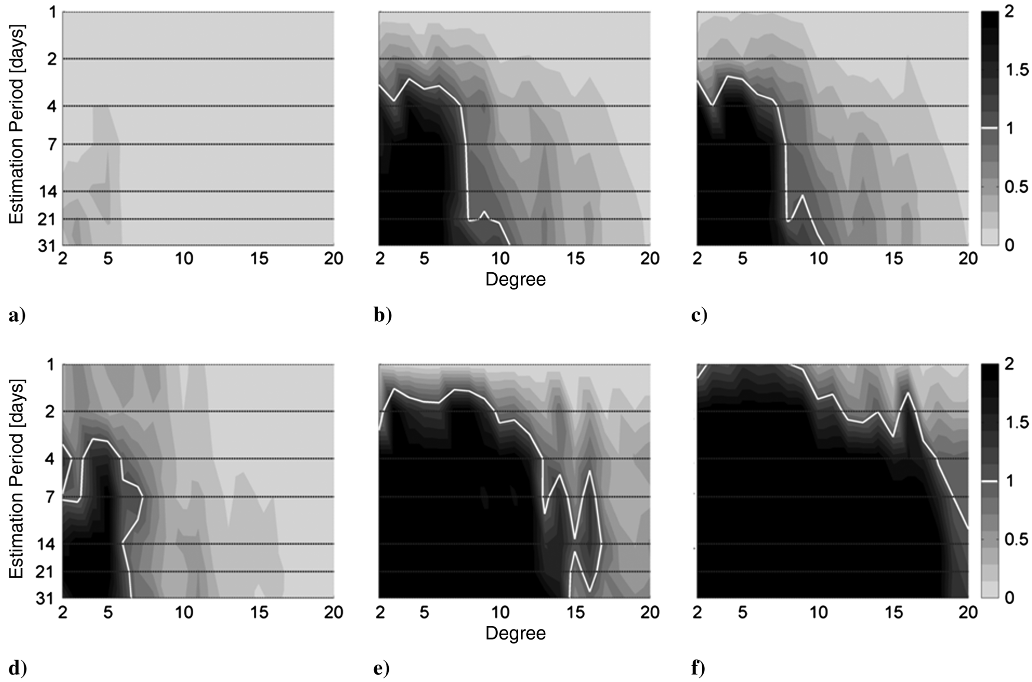

Fig. 8 Signal-to-noise ratios for the following simulated solutions: a and d) F3C/CHAMP, b and e) GRACE, and c and f) combined, using both high-noise (a–c) and low-noise (d–f) scenarios.

accuracy (~2–3 cm), such as that which has already been achieved with CHAMP [10], and which is predicted for Iridium NEXT [17]. Consequently, while the 2–3 cm orbit accuracy is possible for some satellites, the publicly available orbits available from CDAAC are currently not this accurate, and they likely explain the difference in solution quality at the higher degrees. A best fit curve of the simulated high-noise F3C/CHAMP constellation to the real-data curve shows a factor 3.33 difference in the assumed high-noise scenario, which is consistent with the orbit accuracy difference between the high-noise assumption and the 10 cm actual noise level. Future efforts will focus on improving the orbit determination of the F3C satellites, but the fact that the differences can be mostly explained gives confidence to the accuracy of the simulations. The low-noise simulated case shows that it would take a significant improvement in positioning accuracy in order to allow the seven satellites of the F3C/CHAMP constellation to make a noticeable contribution to the solution. If achieved, however, it shows that even a small number of satellites could improve the determination of time-variable gravity field models at the lower degrees.

There are, of course, several items that complicate the interpretation of the real-data and simulated cases. One example is that the simulations only made use of the AOD product for its time-variable signal, which includes only ocean and atmospheric effects. The real-data solutions are affected by a number of other short-term signals not included in the AOD signal, such as continental hydrology, which can have signals on the same order of magnitude as the AOD signals. The treatment of these additional sources of signal variation will need to be considered in future analysis, especially if the orbit accuracies improve to the point where these signals can be clearly observed, as in the low-noise simulated case. For the current study, the exclusion of such additional sources do not impact the overall conclusions of the analysis done due to the low number of satellites in the F3C/CHAMP constellation and their associated orbit accuracies.

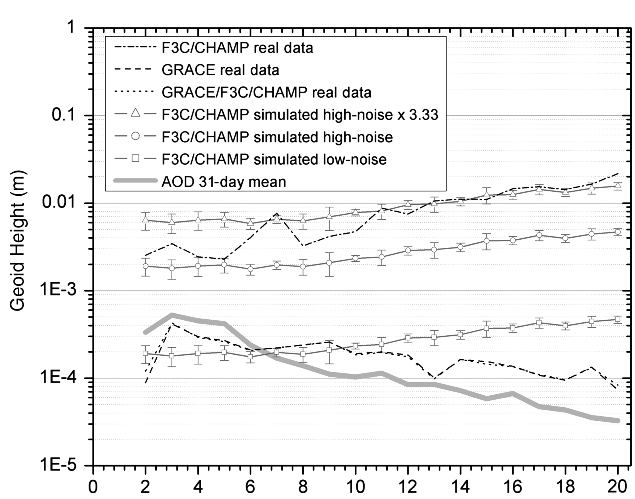

Fig. 9 Comparison of real-data solutions with those predicted from simulation, in terms of square-root degree variances (units of geoid height).

V. Conclusions

Several important conclusions can be drawn from the analysis of this study. The first of these conclusions addresses the question of whether a constellation of nondedicated satellites (i.e., satellites not specifically designed for gravity field determination) can improve our knowledge of Earth's time-variable gravity field. The simulations performed using the proposed Iridium NEXT constellation demonstrated that such nondedicated satellite constellations should be able to observe the large-scale (greater than 1000 km) short-term (i.e., less than one month) gravity signals accurately. Such a constellation would observe a spectrum of the gravity field to which GRACE, a dedicated gravity field mission, is inherently insensitive; the incorporation of the constellation data would improve the overall quality of the time-variable gravity field models produced compared with a GRACE-only solution. The temporal aliasing problem with GRACE is one that can only be solved with the addition of more satellites, and the complementary data that the constellations would provide represent an opportunity to help reduce this error. This option is especially attractive considering that the constellation approach could potentially rely entirely on existing (or future planned) satellites, making such improvements available for a relatively small amount of extra effort, and would not impose any restrictions to the development of a potential GRACE follow-on mission. Related to this is the fact that the gravity measurements derived from these constellations may provide one of the only sources of continuous global time-variable gravity field data in the event there is a gap in coverage between the GRACE and GRACE follow-on missions.

Should the Iridium NEXT project fail to materialize, other suitable constellations are currently under development. Independent of these, a similar constellation could conceivably be developed through the use of low-cost mini-satellites or microsatellites equipped with a GPS receiver. If all that is required are accurate GPS accelerations, then a series of small cannonball-style satellites could periodically be launched in support of a much more complex and expensive dedicated GRACE follow-on mission. These smaller satellites would ideally be inexpensive to build and launch (no dedicated launches required) and would likely have short lifetimes, enabling replacements with improved technology.

The simulations that support these conclusions included error sources representative of the current GRACE and CHAMP missions. In an effort to validate the errors assumed in the simulations, a comparison with mission data collected from the F3C constellation was conducted. One of the results from this real-data processing was that the current level of precise orbit determination of the F3C satellites in the publicly available orbit data is currently insufficient to observe short-term (i.e., less than one month) gravity signals, such as those from oceanic, hydrologic, and atmospheric variations; however, the real-data processing of these orbit data compared favorably with the simulated results, particularly at the lower spherical harmonic degrees and orders. The agreement provides a certain degree of confidence in the simulated results shown for the Iridium NEXT constellation. The analysis also made clear that the extent to which these constellations can help depends largely on the quality of the computed orbits. To achieve improved orbits for future constellations of nondedicated satellites would likely require, in addition to GPS receiver quality, more precise satellite attitude information, the reduction of GPS errors caused by multipath, as well as the more accurate determination of time-varying shifts in the satellite's center of mass (e.g., due to rotating solar panels). Overall, however, the results of the simulations presented here were encouraging, and they highlight the notion that a GRACE follow-on mission could potentially benefit from a supporting constellation of nondedicated satellites.

Acknowledgments

For some components of this study, use was made of the computing resources at Stichting Academisch Rekencentrum Amsterdam (SARA) Reken-en Netwerkdiensten, in the framework of grant SH-053, which was provided by the Stichting Nationale Computerfaciliteiten (NCF). Orbit data for the Formosa Satellite Mission No. 3/Constellation Observing System for Meteorology, Ionosphere, and Climate (F3C) and Challenging Minisatellite Payload (CHAMP) missions was provided by the University Corporation for Atmospheric Research Constellation (UCAR) Observing System for Meteorology, Ionosphere, and Climate (COSMIC) Data Analysis and Archive Center. We would also like to acknowledge the three anonymous reviewers whose comments and suggestions helped to strengthen the paper.

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C. Kluever
Associate Editor